Awareness, Negation and Logical Omniscience

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Abstract

General Epistemic Logics suffer from the problem of logical omniscience, which is that an agent's knowledge and beliefs are closed under implication. There have been many attempts to solve the problem of logical omniscience. However, according to our intuition, sometimes an agent's knowledge and beliefs are indeed closed under implication. Based on the notion of awareness, we introduce two kinds of negations: *general negation* and *strong negation.* Moreover, four kinds of implications, *general implication, strong implication, weak implication,* and *semi-strong implication,* are introduced to correspond with the two kinds of negations. In our logics of regular awareness, explicit beliefs are not necessarily closed under general implication, which means that agents are not logically omniscient. However, explicit beliefs are closed under strong implication and semi-strong implication, which captures an intuitive closure property of beliefs.

'Not' is not not.
- C. J. Date

1 Introduction

As is well known, general epistemic logics, namely, logics of knowledge and belief, suffer from the problem of logical omniscience. The so-called *logical omniscience problem* is that an agent's knowledge and beliefs are closed under logical implication. There have been many attempts to solve the problem of logical omniscience[3, 6, 7, 11, 12, 13, 18, 19]. Some approaches are syntactic, others focus on the refinement of possible world semantics. The general goal is to offer a formalism by which an agent's knowledge and beliefs need not to be closed under logical implication.

However, according to our intuition, sometimes an agent' knowledge and belief are indeed closed under implication. Of course, it is often pointed out that the logical closure of an agent's knowledge and beliefs largely depends on the pragmatics, which seems to be difficult to handle on modeling level. If an agent's knowledge and beliefs are not necessarily closed under implication, however, the question is what roles the notion of implication is playing in reasoning about knowledge and beliefs. Whether there exist other kinds of implication which are intuitive and avoid logical omniscience remains an interesting problem.

In this paper, first of all, we reexamine the problem of logical omniscience, and discuss proposed strategies to handle with the problem. In [10], Hintikka asserts that the *only* reasonable way of solving th problem of logical omniscience is to countenance worlds that are epistemically possible but not logically possible. However, in [7], Fagin and Halpern present some awareness logics, and claim that in the logics agents are not logically omniscient. Based on the analysis of the existing proposed approaches to solve the problem of logical omniscience, we point out that, as a matter of fact, the existing approaches focus on two different perspectives. Some approaches focus on logically possible beliefs to solve the problem, whereas others focus on actual beliefs, which have close relationship with certain psychological notions.

Secondly, based on the analysis of Fagin and Halpern's general awareness, we distinguish some kinds of awareness: *awareness by perception, awareness by computation, indirect awareness,* and *system awareness*. Then, we offer a regular awareness logic. In the logic, two kinds of negations, *general negation and strong negation*, are introduced. Moreover, four kinds of implications, *general implication, strong implication, weak implication* and *semi-strong implication*, are introduced to correspond with the two kinds of negations. In our logics of regular awareness, explicit beliefs are not necessarily closed under general implication. But, explicit beliefs are closed under strong implication and semi-strong implication. Furthermore, the logical closure property does not depend on applications but on awareness, which suggests that it is possible to give an intuitive logical closure interpretation that solves the problem of logical omniscience.

2 Epistemic Logic and Logical Omniscience

2.1 The Classical Kripke Model for Knowledge and Belief

Possible worlds semantics was first proposed by Hintikka [9] for models of epistemic logic, the logic of knowledge and belief. The intuitive idea beyond possible worlds semantics is that besides the true states of affairs, there are a number of other possible worlds, or states. Some of those possible worlds may be indistinguishable to an agent from the true world. An agent is said to know a fact φ if φ is true in all the states he thinks possible.

In this section we briefly review the possible worlds semantics for knowledge and belief. Suppose we consider a logic system concerning n agents, say a set $\mathbf{An} = \{i, j, k, ...\}$ of agents, and we have a set Φ_0 of primitive propositions about which we wish to reason. In order to formalize the reasoning about knowledge and belief, we use a modal propositional logic, which consists of the standard connectives such as $\wedge, \vee$,and$\neg$, and some modal operators $L_i, L_j, ...,$. A formula such as $L_i\varphi$ is to be read as 'agent i believes φ.'

We give semantics to these formulas by means of Kripke structures, which formalize the intuitions behind possible worlds. A Kripke structure for knowledge for n agents is a tuple $(S, \pi, \mathcal{L})$, where S is a set of possible worlds, $\pi(p, s)$ is a truth assignment to the primitive propositions of Φ_0 for each possible world $s \in S$, and $\mathcal{L} : \mathbf{An} \to 2^{S\times S}$ specifies n binary accessibility relations on S. For a knowledge system, the binary relations are equivalence relations. For a belief system, the relations are serial, transitive, and Euclidean. A relation R is *serial* if for each $s \in S$ there is some $t \in S$ such that (s,t) $\in$ R; R is *transitive* if (s, u) $\in$ R whenever (s, t) $\in$ R and (t, u) $\in$ R; R is *Euclidean* if (t, u) $\in$ R whenever (s, t) $\in$ R and (s, u) $\in$ R.

We now assign truth values to formulas at a possible world in a structure. We write $M, s \models \varphi$ if the formula φ is true at possible world s in structure M.

$M, s \models p$, where p is a primitive proposition, iff $\pi(p, s) = true$
$M, s \models \neg\varphi$ iff $M, s \not\models \varphi$,
$M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$,
$M, s \models L_i\varphi$ iff $M, t \models \varphi$ for all t such that $(s, t) \in \mathcal{L}(i)$.

We say a formula φ is *valid in structure M* if $M, s \models \varphi$ for all possible worlds s in M; φ is *satisfiable in M* if $M, s \models \varphi$ for some possible worlds in M. We say φ is *valid* if it is valid in all structures; φ is *satisfiable* if it is satisfiable in some Kripke structure.

The logic of belief above is characterized by the following axiom system, called weak **S5** or **KD45**.

(L1) All instances of propositional tautologies.
(L2) $L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \rightarrow L_i\psi$.
(L3) $\neg L_i(\mathbf{false})$.
(L4) $L_i\varphi \rightarrow L_iL_i\varphi$.
(L5) $\neg L_i\varphi \rightarrow L_i\neg L_i\varphi$.
(R1) $\vdash \varphi, \vdash \varphi \rightarrow \psi \Rightarrow \vdash \psi$.
(R2) $\vdash \varphi \Rightarrow \vdash L_i\varphi$.

(L1) and (R1) hold from propositional logic. (L2) means that an agent's belief is closed under implication, (L3) says that an agent never believe things that are false. The axiom is generally taken to distinguish belief from knowledge. For a knowledge system, (L3) is replaced by a stronger axiom (L3') $L_i\varphi \rightarrow \varphi$, which says that an agent only knows things that are true. (L4) and (L5) are axioms of introspection, which mean that each agent has complete knowledge about his belief set.

2.2 The Problem of Logical Omniscience

Possible world semantics for knowledge and belief does not seem to be an appropriate theory for modelling human reasoning, because they suffer from *the problem of logical omniscience.* An agent is *logical omniscient* if, whenever he believes all of the formulas in a set Ψ, and Ψ logically implies the formula φ, then the agent also believes φ. It is well known that humans, or even computers, are not such perfect reasoners, because thay are generally time and resource limited. In other words, these epistemic logics capture logically possible knowledge and beliefs instead of the agents' actual knowledge and beliefs.

In order to study the problem restrictly, first of all, we would like to draw a distinction between logically possible beliefs and actual beliefs. We call the former *possible beliefs*, and the latter *actual beliefs*. In epistemic logics, we encounter the notions of *explicit belief* and *implicit belief*. Explicit beliefs are those beliefs an agent actually has, that is, his actual beliefs[1], whereas implicit beliefs consists of all of the logical consequences of an

[1]When implicit beliefs are studied, we call actual belief *explicit belief*.

agent's explicit beliefs.[2]

Needless to say, in applications of epistemic logics, especially in artificial intelligence and knowledge engineering, actual belief plays an important role in reasoning about knowledge. However, actual belief seems to be elusive since it may be effected by many sources. These sources [3] are:

1. Awareness
   As Fagin and Halpern in [7] point out, one cannot say that one knows or doesn't know about p if p is a concept he is completely unaware of. In other words, one cannot have actual beliefs [4] about formulas one is not aware of.[5]

2. Resource-bounded
   An agent may have no actual beliefs since he may lack the computational resources.

3. Little importance.
   An agent may not have certain beliefs because of their little importance.

4. Attention entrenchment.
   An agent may pay no attentions to some formulas so that he lacks the actual beliefs, which may be because the beliefs are unimportant, or because the agent does not focus on all relevant issues simultaneously.

5. Prejudices
   An agent may fail to have some actual beliefs because of his prejudices.

Therefore, actual belief seems to be a psychological notion since they have a close relationship with the psychological notions such as awareness, attention, etc., even 'little importance' can be interpreted as irrelevance with agent's intentions and goals. Realistic actual belief seems to be unrealizable. However, approximating actual beliefs is possible. In order to capture a more realistic representation of human reasoning, there have been various attempts to deal with this problem.

3 Approaches Handling With Logical Omniscience

3.1 General Strategy

To solve the problem of logical omniscience is to capture realistic belief consequences. There exist two main strategies to handle with the problem. One strategy is what we call the *logical strategy.* The main goal of logical strategy is to avoid some logical closure for epistemic modal operator. The other strategy is the *psychological strategy*, in which some psychological functions such as awareness are introduced to capture a realistic belief. However, according to our opinion, what is captured by the logical strategy is at most

[2]Note that the notion of implicit belief is different from that of possible belief, because possible belief may have no relationship with actual belief whatsoever.

[3]In [7], Fagin and Halpern list (1),(2) and (4)

[4]In [7], Fagin and Halpern call it explicit belief

[5]Strictly speaking, unawareness does not necessarily mean no actual beliefs. We shall discuss the problem in the subsection 'Indirect Awareness'.

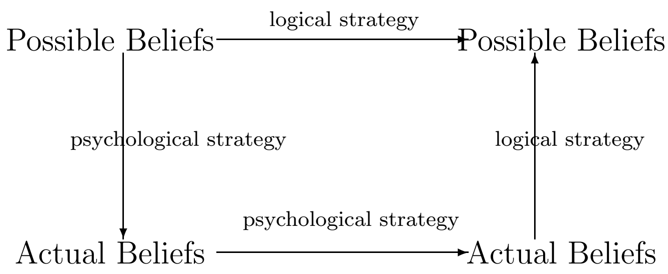

Figure 1: General Strategies

the more realistic possible belief instead of actual belief. Because, as mentioned above, we consider the actual beliefs as a psychological notion, which can be captured only by psychological strategy.

Now, based on the different starting points, approaches capturing realistic belief consequence can be categorized into the following four approach: *approaches from possible beliefs to possible beliefs, approaches from possible beliefs to actual beliefs, approaches from actual beliefs to possible beliefs,* and *approaches from actual beliefs to actual beliefs.* Intuitively, approaches from possible belief to possible beliefs derive more realistic possible beliefs from logically possible belief premises. The others are analogous

3.2 Approach I: From Possible Beliefs to Possible Beliefs

This approach focuses on the invalidation of some logical closure by a logical strategy. However, avoiding logical closure does not necessarily mean capturing realistic actual beliefs. Therefore, they remain to be called possible beliefs.

Formally, we can formalize the closure properties as follows. Let Ψ_K be a set of formulas for an epistemic modal operator K. For a semantics model M, the modal operator K is said to be:

(C1) *closed under implication,*
if $\varphi \in \Psi_K$, and if $\varphi \to \psi \in \Psi_K$, then $\psi \in \Psi_K$.
(C2) *closed under conjunction,*
if $\varphi \in \Psi_K$, and $\psi \in \Psi_K$, then $\varphi \wedge \psi \in \Psi_k$.
(C3) *decomposable under conjunction*
if $\varphi \wedge \psi \in \Psi_K$, then $\varphi \in \Psi_K$, and $\psi \in \Psi_K$
(C4) *closed under axioms of logical theory T,*
if φ is an axiom of some logical theory T, then $\varphi \in \Psi_K$.
(C5) *closed under valid formula,*
if φ is a tautology, then $\varphi \in \Psi_K$.
(C6) *closed under valid implication,*

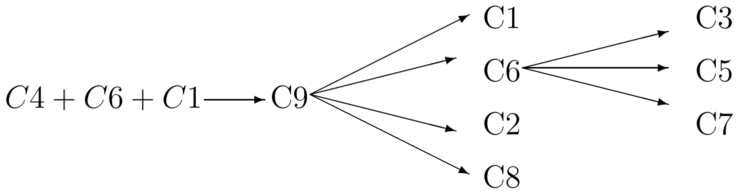

Figure 2: Relationships between Closure Conditions

if $\varphi \in \Psi_K$, and if $\varphi \rightarrow \psi$ is valid, then $\psi \in \Psi_K$.
(C7) *closed under logical equivalence,*
if $\varphi \in \Psi_K$, and φ is logically equivalent to ψ, then $\psi \in \Psi_K$.
(C8) *closed under substitution,*
if $\varphi \in \Psi_K$, then $\varphi\theta \in \Psi_K$ for any substitution θ.
(C9) *logical omniscience,*
if Ψ_K logically implies φ, then $\varphi \in \Psi_K$.

There exist at least the following deductive relationships among those closure conditions above.
(a) $C1 + C5 \rightarrow C6$
(b) $C4 + C8 + C1 \rightarrow C9$
(c) $C4 \rightarrow C5$
(d) $C6 \rightarrow C3 + C5 + C7$
(e) $C9 \rightarrow C1 + C6 + C2 + C8$

Generally, the fewer closures are implied, the more acceptable the condition is. From the relationships between closure conditions above, we know that (C1), (C6) and (C2) play an important part in the problem of logical omniscience. The existing approaches mainly focus on those three closure properties.

Claim 3.1 *In General Epistemic Logics, beliefs are closed under implication, valid implication and conjunction.*

There are some proposals which introduces the notion of *non-classical worlds* in the semantics to solve the problem of logical omniscience. *Non-classical worlds* are worlds in which not all valid formulas need be true. Moreover, in non-classical worlds some inconsistent formulas may be true, hence they are called *impossible worlds* or *nonstandard worlds.*

In [18], Levesque first proposed the notions of implicit and explicit belief. Formally, Levesque uses two modal operators B and L to stand for explicit belief and implicit belief respectively. A *structure for implicit and explicit belief* is a tuple M=(S,$\mathcal{B}$, T, F), where S is a set of situations, $\mathcal{B}$ is a subset of S, and T, F: $\Psi_0 \rightarrow 2^S$. Intuitively, T(p) consists of all situations that support the truth of p, whereas F(p) consists of all situations that support the falsity of p. Obviously, in a situation, a proposition may be true, false, both, or neither. Situations which supports neither the truth nor falsity of some primitive proposition are called *partial situations.* An *incoherent situation* is the situation which supports both the truth and falsity of some primitive propositions.

A *complete situation*, or a possible world, is one that supports either the truth or falsity for every primitive proposition and is not incoherent. A complete situation is *compatible* with a situation s' if $s' \in$ T(p) implies $s \in$ T(p), and $s' \in$ F(p) implies $s \in$ F(p), for each primitive proposition p. $\mathcal{B}^*$ stands for the set of all complete situations in S compatible with some situations in B.

Now, we can define the *support relations* $\models_T$ *and* $\models_F$ between situations and formulas as follows:

$M, s \models_T p$, where p is a primitive proposition, iff $s \in T(p)$,
$M, s \models_F p$, where p is a primitive proposition, iff $s \in F(p)$;
$M, s \models_T \sim \varphi$ iff $M, s \models_F \varphi$,
$M, s \models_F \sim \varphi$ iff $M, s \models_T \varphi$;
$M, s \models_T \varphi_1 \wedge \varphi_2$ iff $M, s \models_T \varphi_1$ and $M, s \models_T \varphi_2$,
$M, s \models_T \varphi_1 \wedge \varphi_2$ iff $M, s \models_F \varphi_1$ or $M, s \models_F \varphi_2$;
$M, s \models_T B\varphi$ iff $M, t \models_T \varphi$ for all $t \in \mathcal{B}$,
$M, s \models_F B\varphi$ iff $M, s \not\models_T B\varphi$;
$M, s \models_T L\varphi$ iff $M, t \models_T \varphi$ for all $t \in \mathcal{B}^*$,
$M, s \models_F L\varphi$ iff $M, s \not\models_T L\varphi$.

From the definitions above, it is ease to see that explicit belief implies implicit, namely, the following axiom holds:
$\models (B\varphi \rightarrow L\varphi)$.
Moreover, although implicit belief is closed under implication and valid implication, explicit belief does not suffer from the problem of logical omniscience .

Claim 3.2 *In Levesque's explicit and implicit beliefs logic, explicit beliefs are closed and decomposable under conjunction, but they are neither closed under implication, nor closed under valid implication.*

As Levesque points out, the following axiom is valid in Levesque's semantics:
$B\varphi \wedge B(\varphi \rightarrow \psi) \rightarrow B(\psi \vee (\varphi \wedge \neg\varphi))$
This means that either the agent's beliefs are closed under implication, or else some situation he believes possible is incoherent. Imagining an agent could consider an incoherent situation possible is generally against our intuitions. Also, Levesque' explicit and implicit logic suffers from a critical representation problem since the language is restricted to formulas where no B or L appears within the scope of another.

3.3 Approach II: From Actual Beliefs to Possible Beliefs

Approaches from actual beliefs to possible beliefs, often called generally syntactic approaches, describe an agent's original actual beliefs by a set of formulas, called the *base beliefs set*, and obtain the logical consequences of the base beliefs set by using some logically incomplete deduction rules.

In [13], Konolige presents a *Deductive Belief System,* in which an agent's beliefs are described as a set of sentences in some formal language, together with a deductive process for deriving consequence of those beliefs. In Konolige's deductive belief system, the general model of deduction is a block tableau sequent system. A block tableau system τ consists of a set of axioms and deduction rules. Konolige's Deductive beliefs model can account for the effect of resource limitations on deriving consequences of the base set. As a consequence, an agent need not believe all the the logical consequences of his beliefs.

However, syntactic approaches are generally difficult for analyzing the properties of knowledge and belief, since knowledge and beliefs are simply represented by an arbitrary set of formulas. For artificial agents such as robots, computers, or knowledge-bases, deduction models of beliefs may be reasonable. However, for rational agents such as humans, even intelligent artificial agents, beliefs obtained by deduction models still are viewed as logically possible beliefs instead of actual beliefs since in rational reasoning there seems to be no simple logical deduction closure for their actual beliefs at all.

3.4 Approach III: From Possible Beliefs to Actual Beliefs

In [7], Fagin and Halpern point out that an agent's lack of knowledge of valid formulas is not due to incoherent situations, but is rather due to the lack of "awareness" on the part of the agent of some primitive propositions, and similar reasons hold for the lack of closure under valid implication.

In order to solve the problem of awareness, Fagin and Halpern offer a solution in which one can decide on a metalevel what formulas an agent is supposed to be aware of. They provide a logic of general awareness, which can be viewed as an approach which combines the syntactic approaches and nonclassical worlds approaches.

In Fagin and Halpern's general awareness logic, in addition to the modal operator B_i and L_i of Levesque's Logic, they also use a modal operator A_i for each agent i. They give the formula $A_i\varphi$ a number of interpretations: " i is aware of φ," "i is able to figure out the truth of φ," or even in the cases of knowledge bases," agent i is able to compute the truth of φ within time T."

Supposed we have a set $\mathbf{A_n}$ of agents and a set Φ_0 of primitive propositions, A *Kripke structure for general awareness*[6] is a tuple:

$M = (S, \pi, \mathcal{L}, \mathcal{A})$
where S is set of states,
$\pi(s, \cdot)$ is a truth assignment for each state $s \in S$,
$\mathcal{L} : \mathbf{A_n} \to 2^{S\times S}$, which consists of n serial, transitive, Euclidean relations on S,
$\mathcal{A} : \mathbf{A_n} \times S \to 2^{\Phi}$

$M, s \models \mathbf{true}$,
$M, s \models p$, where p is a primitive proposition, iff $\pi(s, p)$ =true,
$M, s \models \neg\varphi$ iff $M, s \not\models \varphi$,
$M, s \models \varphi_1 \wedge \varphi_2$ iff $M, s \models \varphi_1$ and $M, s \models \varphi_2$,
$M, s \models A_i\varphi$ iff $\varphi \in \mathcal{A}(i, s)$,
$M, s \models L_i\varphi$ iff $M, t \models \varphi$ for all t such that $(s, t) \in \mathcal{L}(i)$,
$M, s \models B_i\varphi$ iff $\varphi \in \mathcal{A}(i, s)$ and $M, t \models \varphi$ for all t such that $(s, t) \in \mathcal{L}(i)$.

Fagin and Halpern claim that their general awareness logic has the property that agents are not logically omniscient, and the logic is more suitable than traditional logics for modelling beliefs of humans (or machines) with limited reasoning capabilities. However, in [10], Hintikka asserts that the *only* reasonable way of solving the problem of logical omniscience is to countenance worlds that are epistemically possible but not logically possible. Now, a contradiction is avoided, thanks to the distinction between actual belief

[6]Here the notations are different from Fagin and Halpern's original ones because we would like to keep a notational consistency.

and possible belief. That is because Hintikka's assertion refers to the possible beliefs, whereas Fagin and Halpern's assertion refers to actual belief, which is a psychological notion. Therefore, we said that general awareness logic is an approach capturing actual beliefs. In general, the awareness in the logic can be viewed as a complex psychological function which integrates other relevant psychological and computational factors such as attention, prejudices, reasoning capabilities, etc. In the following sections, we intend to study the problem of logical omniscience by an extension of the awareness function.

4 Awareness

In order to capture a more intuitive understanding of the notion of awareness, it is necessary to make a detailed analysis on awareness. There seem to exist many kinds of interpretations of the notion of awareness.

4.1 Awareness by perception

A simple psychological interpretation is *awareness by perception*, which says that to be aware of something is to perceive something. Awareness of a compound is generally[7] built up from the awareness of its parts, namely, the perception of its parts. A suitable semantics approach to formalize the awareness by perception seems to be situation semantics which is proposed by Jon Barwise and John Perry in [1].

4.2 Awareness by computation

Another interpretation about awareness is awareness by computation, which means that to be aware of something is to be able to figure out the truth of that by some special deduction rules or approaches. In other words, non-awareness of something can be interpreted as failure of computation of the truth. That may be because the agent's resources are limited or something else. From the computational point of view, as Konolige suggested in [15], there are two possible approaches that would fit into the awareness framework:

1. *Awareness as filter*
   Agents compute all logical consequences of their beliefs, throwing away those not in the awareness set, perhaps because limitation of memory, perhaps because of agents' prejudices.

2. *Awareness as derivator*
   Agents use a complete logical deduction system to compute consequences of beliefs, but do not pursue those lines of reasoning which require deriving sentences not in the awareness set.

[7] As was pointed out to us by John-Jules Meyer, there are examples where one perceives the whole but not the components.

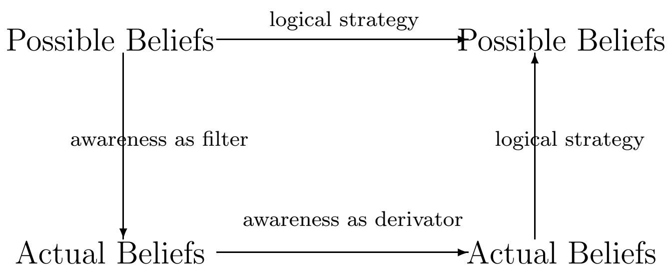

Figure 3: Awareness and Approach

4.3 Indirect awareness

In a multi-agents environment, a specially interesting case is *belief dependence,* which means that some agents may rely on someone else about its beliefs and knowledge. Specially, an agent may be not aware of a formula φ. But he may be aware of the agents who are aware of φ. That case is called *indirect awareness.* In general awareness logic an agent cannot have any explicit belief about some formula φ if he is not aware of φ. However, we argue that unawareness does not necessarily result in the failures of capturing explicit beliefs. For instance, suppose you read a sentence says '*The rabbit is an oryctolagus cuniculus*' in a zoological book. Although you may not be aware of 'oryctolagus cuniculus', you may believe that the rabbit is an oryctolagus cuniculus is true since you generally believe what the author says. Therefore, indirect awareness can be viewed as an intuitive extension of the notion of awareness. In [12], we present some semantics models about belief dependence. In general, the goals of a system of belief dependence are to change all indirect awareness into *direct awareness*, namely, the general notion of awareness.

4.4 System awareness

In reasoning about multi-agents' knowledge and belief, a reasoner may be one of those agents whose knowledge and belief are formalized and reasoned with. However, specially, the reasoner may not be one of those agents but only an observer, or called *super-agent.*

In general awareness logics, we only can formalize on agents' general awareness. It is frequently beneficial to be able to formalize super-agent's awareness itself. We call it *system awareness.* In fact, system awareness set specifies the estimation of a system reasoner about its reasoning capabilities. The notion of system awareness has a close relationship with the notion of "unknown", which is generally introduced in some systems of knowledge and belief, especially, in knowledge bases and the database system with null values. Intuitively, nonawareness means "unknown".

Moreover, in a multi-agents environment, even though a super-agent is one of those agents whose beliefs and knowledge are reasoned with, we should draw a distinctions between awareness of general agent and the awareness of super agent.

5 Negation

In [15], Konolige points out that the problem of logical omniscience results from the introduction of possible worlds in analysis of beliefs. He suggests that the introduction of possible worlds should be rethought. However,the idea of possible worlds indeed has a certain intuitive appeal. A suitable re-examination should focus on the truth conditions of possible world semantics instead of on the introduction of possible worlds.

After the reexamination of the truth conditions of possible worlds semantics for a logic of beliefs,we find that the condition concerning negation, namely $M, s \models \neg\varphi$ iff $M, s \not\models \varphi$, is somewhat against our intuitions. We do not necessarily conclude the negation of a formula φ whenever we could not conclude a formula φ , if we do not know whether that is due to our resource and time limits or due to our nonawareness. On the other hand, there might exist different kinds of negations in our intuitions. Those cases similarly happen in the intuitionistic logics.[4, 8] Moreover, in the studies of null values in database theory, C. J. Date [5] also appeals for the necessities of introducing different kinds of negations.

Based on the notion of awareness, we find that we can present an intuitive formalism about multi-negations in the logics of knowledge and belief, which also can avoid logical omniscience.

Suppose we have a set $\mathbf{A_n}$ of n agents, and a set Ψ_0 of primitive propositions, the language $\mathbf{L}$ is the minimal set of formulas closed the following syntactic rules:

(i) $p \in \Psi_0 \Rightarrow p \in \mathbf{L}$
(ii) $\varphi \in \mathbf{L}, \psi \in \mathbf{L} \Rightarrow \varphi \wedge \psi \in \mathbf{L}$,
(iii) $\varphi \in \mathbf{L} \Rightarrow \neg\varphi \in \mathbf{L}$,
(iv) $\varphi \in \mathbf{L}, i \in \mathbf{A_n} \Rightarrow L_i\varphi \in \mathbf{L}$,
(v) $\varphi \in \mathbf{L}, i \in \mathbf{A_n} \Rightarrow A_i\varphi \in \mathbf{L}$

As far as the semantics structure is concerned, at least, we should make some constraints on awareness sets. First of all, we think that closure under negation,is a reasonable condition since awareness of a formula φ means awareness of the negation $\neg\varphi$. Then, closure under conjunction is an acceptable condition. Furthermore, we suggest that awareness of awareness implies awareness, namely, $A_iA_i\varphi \rightarrow A_i\varphi$, also is an acceptable condition intuitively. However, we do not intend to accept the condition $A_i\varphi \rightarrow A_iA_i\varphi$, because awareness of something does not necessarily result in awareness of the awareness. We call the condition $A_iA_i\varphi \rightarrow A_i\varphi$ *the nested awareness reduction condition*[8]. Finally, we give an interpretation about awareness of beliefs, namely, $A_iL_i\varphi \equiv A_iA_i\varphi$.[9] We call

[8]We agree at the axiom of awareness $A_iA_i\varphi \rightarrow A_i\varphi$. For instance, in awareness by perception, one who can perceive his perception naturally has the perception. But, we generally do not accept the axiom $A_i\varphi \rightarrow A_iA_i\varphi$. We can offer some examples to show that $A_i\varphi \wedge \neg A_iA_i\varphi$ is possible in real life. A vivid, perhaps trivial, example is dreaming. In his dream, one person may dream, or perceive, something such as beautiful flowers. But, he generally has no perception of the perception. In other words, he fails to figure out whether his perception is true or not.(Sometimes psychologists talk about *lucid dreams*, in which dreamer can be aware of his dreaming. However, we do not consider the special situations here.)

[9]This interpretation seems to be somewhat special. Of course, awareness of awareness is not se-

the condition $A_iA_i\varphi \equiv A_iL_i\varphi$ *the belief awareness interpretation.* Moreover, we call the awareness which satisfies the four conditions above *regular awareness.*

Formally, A *Kripke structure with regular awareness* is a tuple $M = (S, \pi, \mathcal{L}, \mathcal{A})$ where, as before, S is a set of states,$\pi(s, \cdot)$ is a truth assignment for each state $s \in S$, and $\mathcal{L} : \mathbf{A_n} \to 2^{S\times S}$, which consists of n binary accessibility relations on S, and for each state s and each agent $i \in \mathbf{A_n}, \mathcal{A}(i, s)$, awareness formula set, is a subset of formulas in the language $\mathbf{L}$, and the structure satisfies the following constraints:
(i) the binary accessibility relations are serial, transitive, and Euclidean, and
(ii) awareness formula sets are closed under negation, and closed under conjunction, and satisfy the nested awareness condition and belief awareness interpretation, namely, $\varphi \in \mathcal{A}(i, s)$ iff $\neg\varphi \in \mathcal{A}(i, s)$, $\varphi \wedge \psi \in \mathcal{A}(i, s)$ iff $\varphi,\ \psi \in \mathcal{A}(i, s)$, and if $A_i\varphi \in \mathcal{A}(i, s)$, then $\varphi \in \mathcal{A}(i, s)$, and $A_i\varphi \in \mathcal{A}(i, s)$ iff $L_i\varphi \in \mathcal{A}(i, s)$

The truth relation $\models$ is defined inductively as follows:
$M, s \models p$, where p is a primitive proposition, iff $\pi(s, p)$ =true,
$M, s \models \neg\varphi$ iff $M, s \not\models \varphi$
$M, s \models \varphi_1 \wedge \varphi_2$ iff $M, s \models \varphi_1 \wedge M, s \models \varphi_2$,
$M, s \models A_i\varphi$ iff $\varphi \in \mathcal{A}(i, s)$
$M, s \models L_i\varphi$ iff $M, t \models \varphi$ for all t such $(s, t) \in \mathcal{L}(i)$.
Also,we define $\wr_i\varphi \equiv \varphi \wedge A_i\varphi$, and $\sim_i \varphi \equiv \wr_i\neg\varphi$.

Now, we have two kinds of negations: $\neg$ and $\sim_i$. Furthermore, we call $\neg$ and $\sim_i$ *general negation* and *strong negation*[10] respectively. Intuitively, $\wr_i\varphi$ is read as "φ is true, and agent i is aware of it", $\sim_i \varphi$ is read as "φ is false, and agent i is aware of it", and $\neg\varphi$ is "φ is generally false".

Propositions 5.1 *(General Negation and Strong Negation)*
(a) $\models \sim_i \neg\varphi \to \varphi$.
(b) $\models \sim_i\sim_i \varphi \to \varphi$.
(c) $\models \sim_i \varphi \to \neg\varphi$.
(d) $\models \varphi \to \neg \sim_i \varphi$.
(e) $\not\models \varphi \to \sim_i\sim_i \varphi$.

It should be noted that $A_i\varphi \wedge (\varphi \equiv \psi) \wedge \neg A_i\psi$ is satisfiable because of the semantics concerning awareness operator. However, if the formula φ is defined as the formula ψ, of course, we have $A_i\varphi \wedge (\varphi \stackrel{def}{=} \psi) \to A_i\psi$.

Propositions 5.2 *(Negation, Awareness, and Certainty)*
(a) $\models \sim_i\sim_i \varphi \to \wr_i\varphi$
(b) $\models \neg \wr_i \varphi \wedge A_i\varphi \to \neg\varphi$
(c) $\models \wr_i(\varphi \wedge \psi) \equiv \wr_i\varphi \wedge \wr_i\psi$
(d) $\models A_i(\varphi \wedge \psi) \equiv A_i\varphi \wedge A_i\psi$
(e) $\models A_i\neg\varphi \equiv A_i\varphi$
(f) $\models A_iA_i\varphi \to A_i\varphi$.
(g) $\models A_iL_i\varphi \to A_i\varphi$.

mantically equal to the awareness of beliefs. However, for the **S5** system, the interpretation is more intuitive because L is interpreted as "know" and awareness of awareness is generally equal to awareness of knowledge.

[10]However, our strong negation here is somewhat different from that in intuitionistic logics.

In [7], Fagin and Halpern define explicit belief by $B_i\varphi \equiv L_i\varphi \wedge A_i\varphi$. Semantically, this means that $M, s \models B_i\varphi$ whenever $\varphi \in \mathcal{A}(i,s)$ and $M, t \models \varphi$ for all t such that $(s,t) \in \mathcal{L}(i)$. However, according to the definition, it is possible that there exists a state t such that $(s,t) \in \mathcal{L}(i)$ and $\varphi \notin \mathcal{A}(i,t)$, which means that agent i may not be aware of φ in a state t though he believes φ is true. Therefore, we would like to make a little modification on Fagin and Halpern's original definition about explicit belief. We define that $B_i\varphi \equiv L_i\varphi \wedge A_i(L_i\varphi)$, namely, $B_i\varphi \equiv \wr_i L_i\varphi$. Meanwhile, we would like to keep Fagin and Halpern's original definition about explicit beliefs. We define that $B_i^{fh}\varphi \equiv L_i\varphi \wedge A_i\varphi$. It is easy to see that $B_i\varphi \to B_i^{fh}\varphi$, but not $B_i^{fh}\varphi \to B_i\varphi$, which means that explicit beliefs by our definition are more restricted beliefs than those obtained by Fagin and Halpern's original definition.

Propositions 5.3 *(Explicit Belief and Implicit Belief)*
(a) $\models B_i\varphi \to L_i\varphi$.
(b) $\models B_i(\varphi \wedge \psi) \equiv B_i\varphi \wedge B_i\psi$.
(c) $\models B_i\neg\varphi \to \neg L_i\varphi$.
(d) $\models \sim_i B_i\varphi \to \sim_i L_i\varphi$.

6 Implication

For implication, we offer two kinds of implications, general implication $\to$ and strong implication $\rightsquigarrow_i$, which respectively correspond with two kinds of negations. *General implication* $\to$ is defined as before, namely $\varphi \to \psi \equiv \neg(\varphi \wedge \neg\psi)$. Whereas *strong implication* $\rightsquigarrow_i$ is defined as $\varphi \rightsquigarrow_i \psi \equiv \sim_i (\varphi \wedge \sim_i \psi)$. Sometimes we need two weakly strong negations, called *weak implication* $\rightharpoondown_i$ and *semi-strong implication* $\hookrightarrow_i$, which are defined as $\varphi \hookrightarrow_i \psi \equiv \sim_i (\varphi \wedge \neg\psi)$ and $\varphi \rightharpoondown_i \psi \equiv \neg(\varphi \wedge \sim_i \psi)$ respectively.

Propositions 6.1 *(General Implication)*
(a) $\models (\varphi \rightsquigarrow_i \psi) \to (\varphi \to \psi) \wedge A_i\varphi \wedge A_i\psi$
Proof:
$\models \varphi \rightsquigarrow_i \psi$
$\equiv \sim_i (\varphi \wedge \sim_i \psi) \equiv \wr_i \neg(\varphi \wedge \sim_i \psi)$
$\equiv \neg(\varphi \wedge \sim_i \psi) \wedge A_i(\varphi \wedge \sim_i \psi)$
$\to \neg(\varphi \wedge \neg\psi \wedge A_i\psi) \wedge A_i\varphi \wedge A_i\psi$
$\to (\varphi \to \psi) \wedge A_i\varphi \wedge A_i\psi$
(b) $\models (\varphi \hookrightarrow_i \psi) \equiv \wr_i(\varphi \to \psi)$.
(c) $\models (\varphi \rightsquigarrow_i \psi) \to (\varphi \hookrightarrow_i \psi)$
(d) $\models (\varphi \hookrightarrow_i \psi) \to (\varphi \rightharpoondown_i \psi)$
(e) $\models (\varphi \hookrightarrow_i \psi) \to (\varphi \to \psi)$
(f) $\models (\varphi \to \psi) \wedge A_i\psi \to (\varphi \rightharpoondown_i \psi)$
(g) $\models (\varphi \rightharpoondown_i \psi) \wedge A_i\psi \to (\varphi \to \psi)$

There is actually a deeper relationship between awareness and the four kinds of implication. It is not hard to show that:

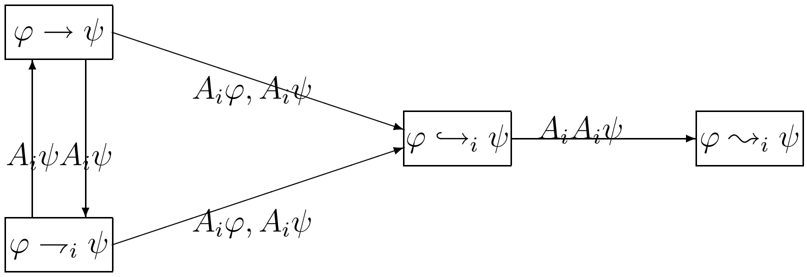

Figure 4: Implications and Awareness

Propositions 6.2 *(Implications and Awareness)*
(a) $\models (\varphi \rightharpoondown_i \psi) \wedge A_i\psi \equiv (\varphi \to \psi) \wedge A_i\psi$.
(b) $\models (\varphi \hookrightarrow_i \psi) \equiv (\varphi \to \psi) \wedge A_i\varphi \wedge A_i\psi$.
(c) $\models (\varphi \rightsquigarrow_i \psi) \equiv (\varphi \to \psi) \wedge A_i\varphi \wedge A_i\psi \wedge A_iA_i\psi$.

The propositions above suggest that it seems to be an alternative approach to start our investigation based on awareness and weak implication instead of general awareness. This is an interesting future work. Moreover, the propositions suggest that implications can be defined more intuitively. In the following sections we select those propositions as the definition of strong implication and semi-strong implication.

Propositions 6.3 *(General Implication and Beliefs)*
(a) $\models L_i\varphi \wedge L_i(\varphi \to \psi) \to L_i\psi$.
(b) $\models B_i^{fh}\varphi \wedge B_i^{fh}(\varphi \to \psi) \to B_i^{fh}\psi$.
(c) $B_i\varphi \wedge B_i(\varphi \to \psi) \wedge \neg B_i\psi$ *is satisfiable.*

From the propositions above, we know that implicit beliefs are closed under general implication, whereas explicit beliefs are not necessarily closed under general implication. Naturally, now, our interest is if explicit beliefs are closed under other implications. We have the following result:

Theorem 6.1 *For regular awareness logics, explicit beliefs are closed under semi-strong implication and strong implication.*

Proof:
$B_i\varphi \wedge B_i(\varphi \hookrightarrow_i \psi)$
$\equiv L_i\varphi \wedge A_iL_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \wedge A_iL_i(\varphi \hookrightarrow_i \psi)$
$\Rightarrow L_i\varphi \wedge A_iL_i\varphi \wedge L_i(\varphi \to \psi) \wedge L_iA_i\varphi \wedge L_iA_i\psi \wedge A_iL_i((\varphi \to \psi) \wedge A_i\varphi \wedge A_i\psi)$
$\Rightarrow L_i\psi \wedge A_i(\varphi \to \psi) \wedge A_iA_i\varphi \wedge A_iA_i\psi$
$\Rightarrow L_i\psi \wedge A_iA_i\psi$
$\Rightarrow L_i\psi \wedge A_iL_i\psi$
$\equiv B_i\psi$

Therefore, explicit beliefs are closed under semi-strong implication. The case of strong implication can be similarly shown. □

As far as other cases are concerned, we also have the following results:

Propositions 6.4 *(Implicit Beliefs and Implication)*
(a) $\models L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \rightarrow L_i\psi$.
(b) $\models L_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \rightarrow L_i\psi$.
(c) $\models L_i\varphi \wedge L_i(\varphi \rightharpoondown_i \psi) \wedge L_iA_i\psi \rightarrow L_i\psi$.
(d) $\models L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \rightharpoondown_i L_i\psi$.
(e) $\models L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \rightharpoondown_i L_i\psi$.
(f) $\models L_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \rightharpoondown_i L_i\psi$.
(g) $\models L_i\varphi \wedge L_i(\varphi \rightharpoondown_i \psi) \wedge L_iA_i\psi \rightharpoondown_i L_i\psi$.

Propositions 6.5 *(Explicit Beliefs and Implication)*
(a) $\models B_i\varphi \wedge B_i(\varphi \rightarrow \psi) \wedge A_iL_i\psi \rightarrow B_i\psi$.
(b) $\models B_i\varphi \wedge B_i(\varphi \rightharpoondown_i \psi) \wedge L_iA_i\psi \wedge A_iL_i\psi \rightarrow B_i\psi$.
(c) $\models B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \rightharpoondown_i B_i\psi$.
Proof:
$\models B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \wedge \sim_i B_i\psi$
$\rightarrow L_i\varphi \wedge A_iL_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \wedge A_iL_i(\varphi \rightsquigarrow_i \psi) \wedge \neg L_i\psi \wedge A_iL_i\psi \wedge A_i\psi$
$\rightarrow L_i\psi \wedge A_iL_i\varphi \wedge A_iL_i(\varphi \rightsquigarrow_i \psi) \wedge \neg L_i\psi \wedge A_iL_i\psi \wedge A_i\psi$
$\rightarrow$ **false**.

Therefore, $\models B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \rightharpoondown_i B_i\psi$.
(d) $\models B_i\varphi \wedge B_i(\varphi \hookrightarrow_i \psi) \rightharpoondown_i B_i\psi$.
(e) $\models B_i\varphi \wedge B_i(\varphi \rightarrow \psi) \rightharpoondown_i B_i\psi$.
(f) $\models B_i\varphi \wedge B_i(\varphi \rightharpoondown_i \psi) \wedge L_iA_i\psi \rightharpoondown_i B_i\psi$.

Propositions 6.6 *(Belief, Implication, and Certainty)*
(a) $\models L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \rightarrow L_i \wr_i \wr_i\psi$
Proof:
$\models L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi)$
$\equiv L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \wedge L_iA_i\varphi \wedge L_iA_i\psi \wedge L_iA_iA_i\psi$
$\rightarrow L_i\psi \wedge L_iA_i\psi \wedge L_iA_i\psi \wedge L_iA_iA_i\psi$
$\rightarrow L_i \wr_i \psi \wedge L_iA_i \wr_i \psi$
$\rightarrow L_i \wr_i \wr_i\psi$.
(b) $\models L_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \rightarrow L_i \wr_i \psi$
(c) $B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \rightarrow \wr_i L_iA_i \wr_i \psi$.
(d) $B_i(\varphi \hookrightarrow_i \psi) \rightarrow B_i^{fh}A_i\psi$
(e) $L_i\varphi \wedge (L_i\varphi \hookrightarrow_i L_i\psi) \rightarrow B_i\psi$.
(f) $L_i\varphi \wedge (L_i\varphi \rightsquigarrow_i L_i\psi) \rightarrow \wr_i B_i\psi$.

Up to now, we have not given any intuitive interpretations about implications, especially, strong implication and semistrong implication. However, as propositions (e) and (f) above suggest, by semi-strong implication one can get explicit beliefs from one's implicit beliefs, whereas by strong implication certain explicit beliefs can be obtained from implicit beliefs, which can be viewed as one of the interpretations. Moreover, we have the following results:

Type	Closure	Additional Condition
$(L \rightarrow\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \rightarrow L_i\psi$	—
$(L \rightsquigarrow\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \rightarrow L_i\psi$	—
$(L \hookrightarrow\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \rightarrow L_i\psi$	—
$(L \neg\!\!\rightarrow\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \neg\!\!\rightarrow_i \psi) \rightarrow L_i\psi$	$L_iA_i\psi$
$(L \rightarrow\neg\!\!\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \rightarrow \psi) \neg\!\!\rightarrow_i L_i\psi$	—
$(L \rightsquigarrow\neg\!\!\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \rightsquigarrow_i \psi) \neg\!\!\rightarrow_i L_i\psi$	—
$(L \hookrightarrow\neg\!\!\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \hookrightarrow_i \psi) \neg\!\!\rightarrow_i L_i\psi$	—
$(L \neg\!\!\rightarrow\neg\!\!\rightarrow)$	$L_i\varphi \wedge L_i(\varphi \neg\!\!\rightarrow_i \psi) \neg\!\!\rightarrow_i L_i\psi$	$L_iA_i\psi$
$(B \rightarrow\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \rightarrow \psi) \rightarrow B_i\psi$	$A_iL_i\psi$
$(B \rightsquigarrow\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \rightarrow B_i\psi$	—
$(B \hookrightarrow\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \hookrightarrow_i \psi) \rightarrow B_i\psi$	—
$(B \neg\!\!\rightarrow\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \neg\!\!\rightarrow_i \psi) \rightarrow B_i\psi$	$LiA_i\psi \wedge A_iL_i\psi$
$(B \rightarrow\neg\!\!\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \rightarrow \psi) \neg\!\!\rightarrow_i B_i\psi$	—
$(B \rightsquigarrow\neg\!\!\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \rightsquigarrow_i \psi) \neg\!\!\rightarrow_i B_i\psi$	—
$(B \hookrightarrow\neg\!\!\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \hookrightarrow_i \psi) \neg\!\!\rightarrow_i B_i\psi$	—
$(B \neg\!\!\rightarrow\neg\!\!\rightarrow)$	$B_i\varphi \wedge B_i(\varphi \neg\!\!\rightarrow_i \psi) \neg\!\!\rightarrow_i B_i\psi$	$LiA_i\psi$

Figure 5: Summaries about Closures

Propositions 6.7 *(Distinction among Implications)*
(a) $\vdash (\varphi \rightarrow \psi) \rightarrow \vdash (L_i\varphi \rightarrow L_i\psi)$.
(b) $\vdash (\varphi \rightsquigarrow_i \psi) \rightarrow \vdash (B_i\varphi \rightarrow B_i\psi)$.
(c) $\vdash (\varphi \hookrightarrow_i \psi) \rightarrow \vdash (B_i\varphi \rightarrow B_i^{fh}\psi)$.
Proof: *(b)* $\vdash \varphi \rightsquigarrow_i \psi, \vdash B_i\varphi$
$\Rightarrow \vdash (\varphi \rightarrow \psi) \wedge A_i\varphi \wedge A_i\psi \wedge A_iA_i\psi, \vdash L_i\varphi$
$\Rightarrow \vdash L_i\psi \wedge A_iA_i\psi \Rightarrow \vdash L_i\psi \wedge A_iL_i\psi \Rightarrow \vdash B_i\psi$.

The above propositions show an intuitive distinction among those implications. Although we have argued that awareness has different interpretations, which depend on the applications, we would like to give an example, in order to give readers a better understanding about those implications and their applications.

Suppose the awareness function is interpreted as perception, that is, awareness by perception, we have the following formulas:
φ: It is raining.
ψ_1: The ground is wet.
ψ_2: The air humidity is up.
ψ_3: The relative humidity is about 70 percent.

If agent i is a child, we generally have:
$(\varphi \rightarrow \psi_1) \wedge A_i\varphi \wedge A_i\psi_1 \wedge A_iA_i\psi_1$.
(Because children can preceive that the ground is wet and know about their perception.)
$(\varphi \rightarrow \psi_2) \wedge A_i\varphi \wedge A_i\psi_2 \wedge \neg A_iA_i\psi_2$.
(Because children can perceive the change of the air humidity, but they generally are not aware of their perception.)
$(\varphi \rightarrow \psi_3) \wedge A_i\varphi \wedge \neg A_i\psi_3 \wedge \neg A_iA_i\psi_3$[11].

[11]Strictly speaking, the relative humidity has not necessarily a close relationship with raining.

(Because children cannot perceive that fact at all.)

In other words, we have the following assertions:

$\varphi \rightsquigarrow_i \psi_1$
$\varphi \hookrightarrow_i \psi_2$
$\varphi \rightarrow \psi_3$

According to the propositions, we know that agent i will have the explicit belief about ψ_1 whenever he has the explicit belief about φ, agent i will have the explicit belief as defined by Fagin and Halpern about ψ_2 whenever he has the explicit belief about φ, and agent i will have the implicit belief about ψ_3 whenever he has the implicit belief about φ.

7 Conclusions

We have investigated the awareness approach in reasoning about knowledge and beliefs, and argue that awareness is an important notion in artificial intelligence and epistemic logics. We also have examined the notion in its relationship with some critical problems in the field. We can summarize it in the following perspectives:

(i) *Awareness and the problem of logical omniscience.*
The awareness approach which is proposed by Fagin and Halpern to solve the problem of logical omniscience, has been subjected by some criticisms [15]. We point out that the awareness approach is a different approach from the general logical strategy to solve the problem. By this new strategy which may integrate some psychological notions we expect to capture a more flexible and realistic knowledge and beliefs system, which can be called systems from actual beliefs to actual beliefs.

(ii) *Awareness and its variants.*
We have discussed some variants of general awareness from different perspectives and approaches: awareness by perception, awareness by computation, awareness as filter, awareness as derivator, indirect awareness, and system awareness. Those notions and approaches provide a promising and interesting research perspectives.

(iii) *Awareness and negation*
Based on the notion of awareness, we have presented two kinds of negations: general negation and strong negation. Moreover, four kinds of implications have introduced to correspond with the two kinds of negations. The relationship between the implication and beliefs, implicit beliefs and explicit beliefs, is examined. The results show that there exist some more intuitive closure properties in regular awareness logics than in general awareness logic.

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